Low Energy Transport Considerations

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Optics

At low energy, it's easiest to use electrostatic optics.

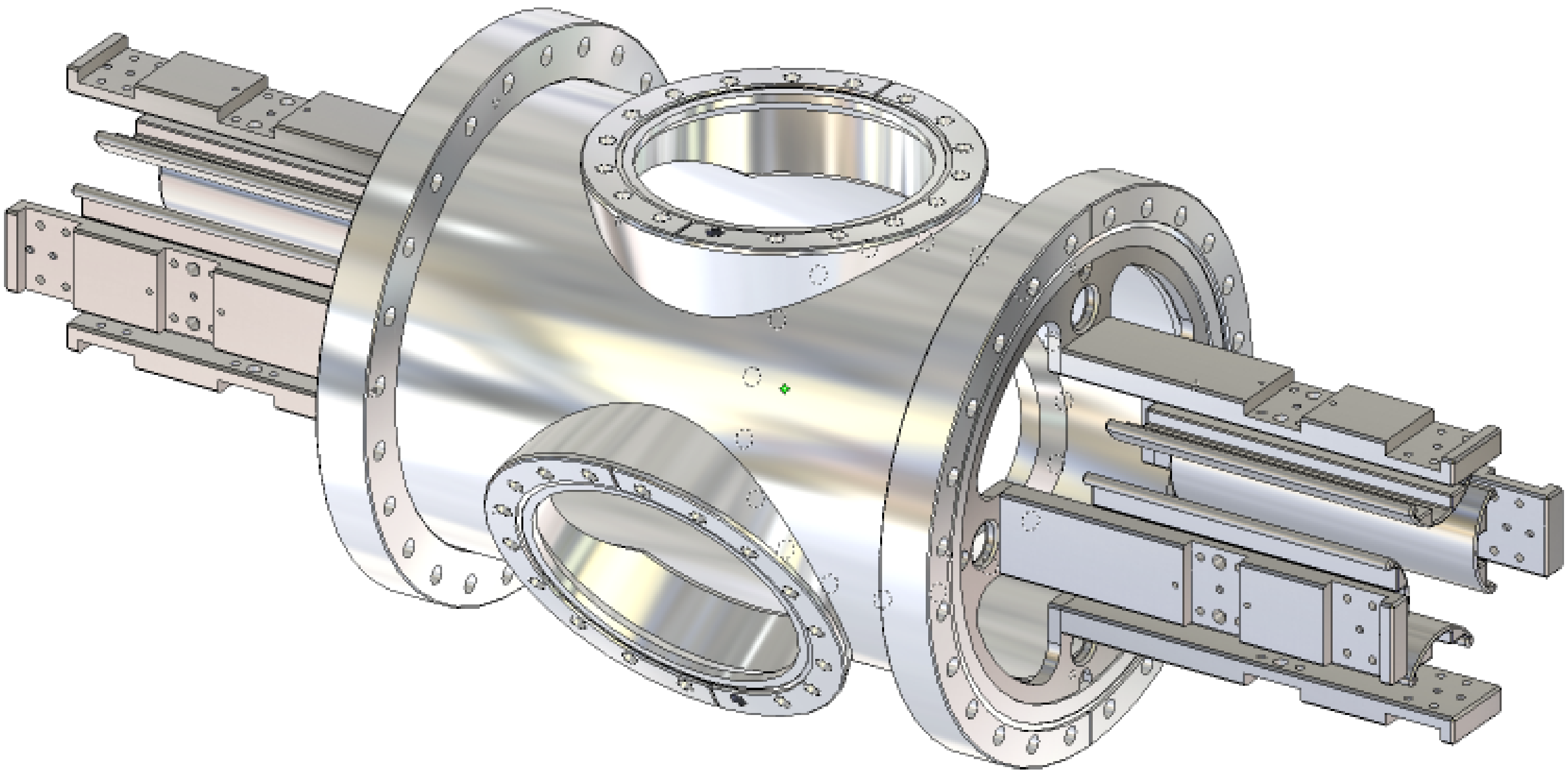
Focusing: Quadrupoles are linear up to aperture limit, but einzel lenses and solenoids are not. Also, quads have much lower voltage while einzel lenses need potentials approx. same as particle energy.

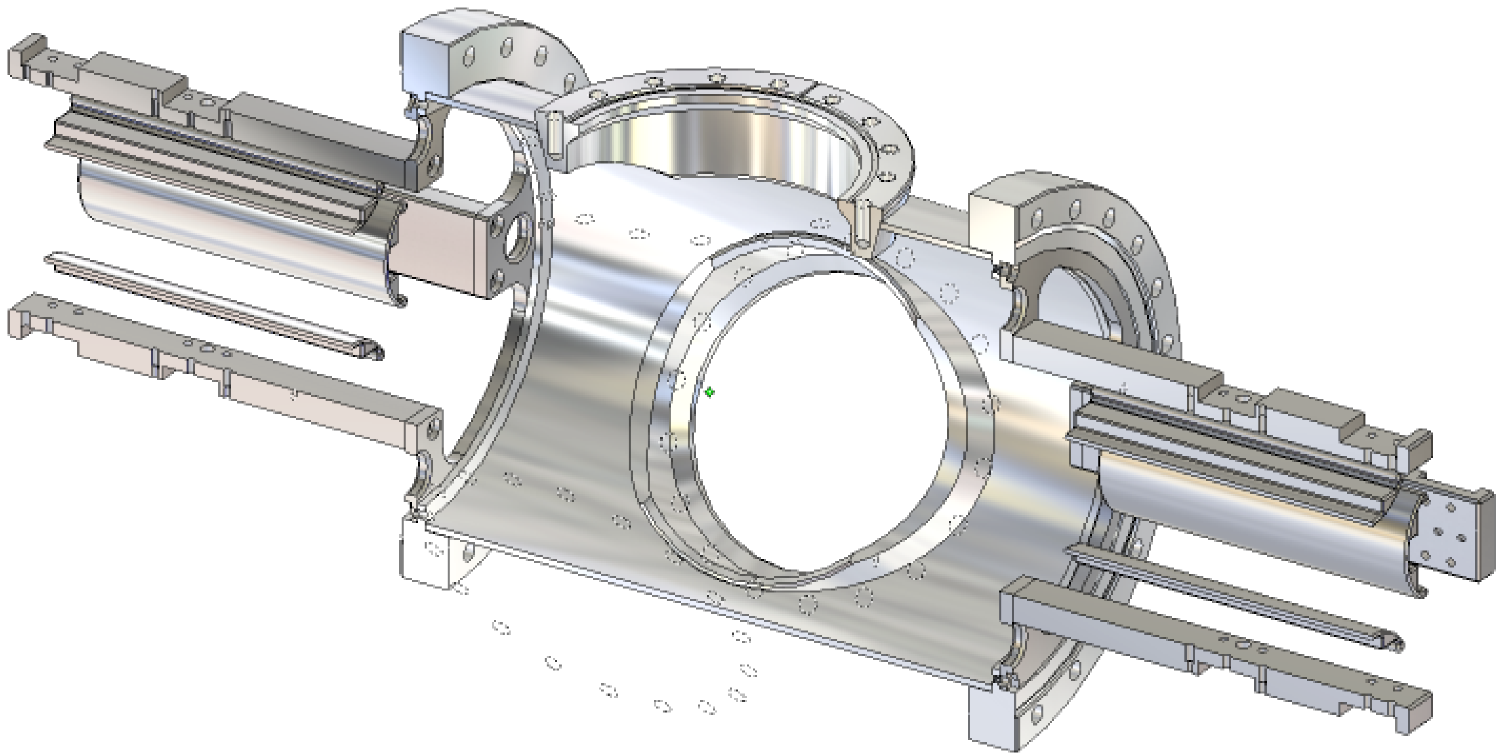
Linear fields $E_x \propto x \Rightarrow V \propto x^2$. Laplace $\Rightarrow V = k(x^2 - y^2)$ or $V = V_0 \frac{x^2-y^2}{a^2}$, where a is aperture radius.

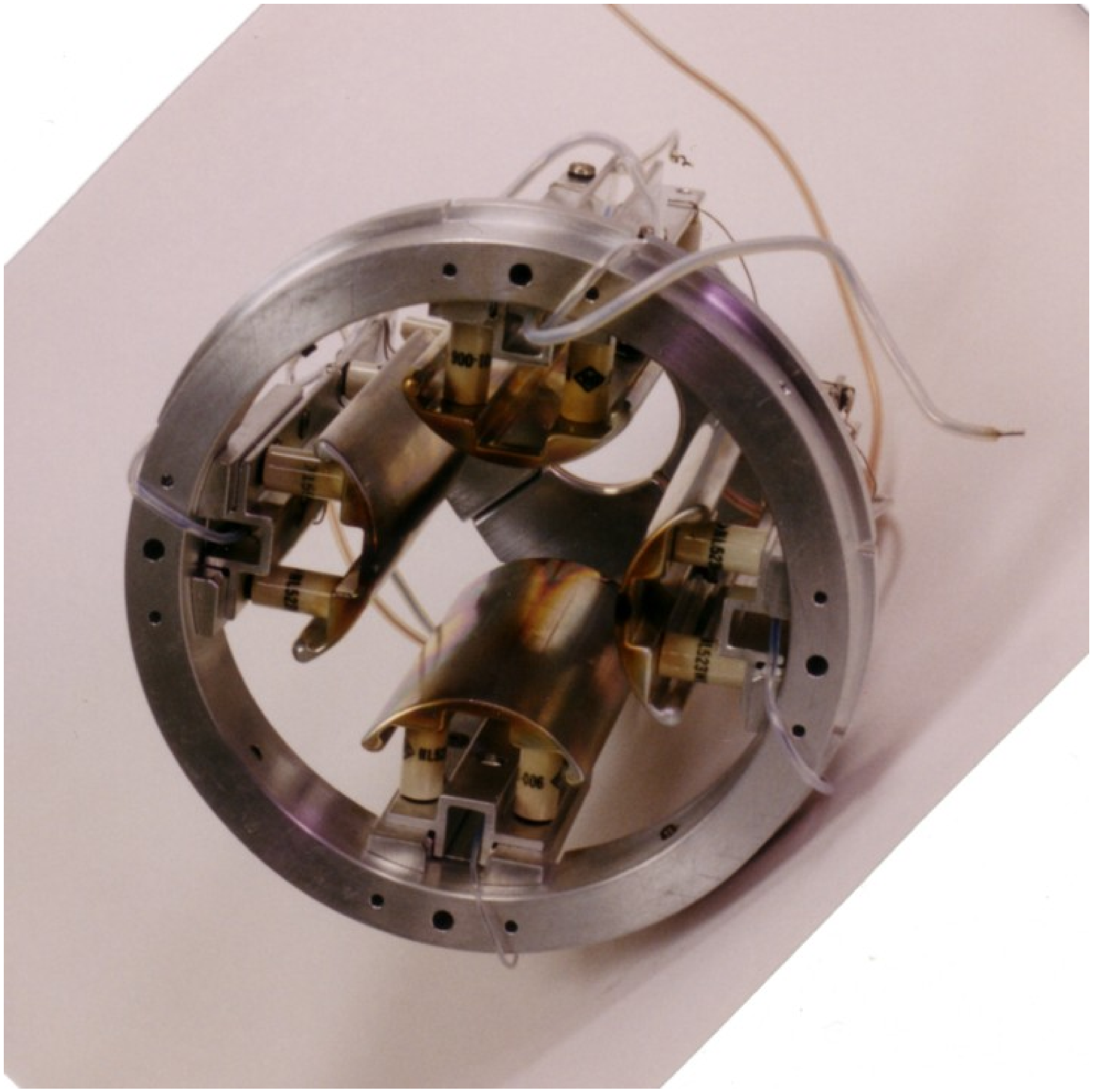
Quad focal length:

$$\frac{1}{f} = \frac{V_0}{V_E} \frac{L}{a^2}$$

V_E is beam energy (per charge) $= \frac{1}{2}mv^2$ non-relativistically, so at 30keV, a quad of length 1 inch, aperture dia. 2 inches ($a = 1$ inch), has a focal length of $f = 10$ inches for an electrode voltage as small as 3 kV.







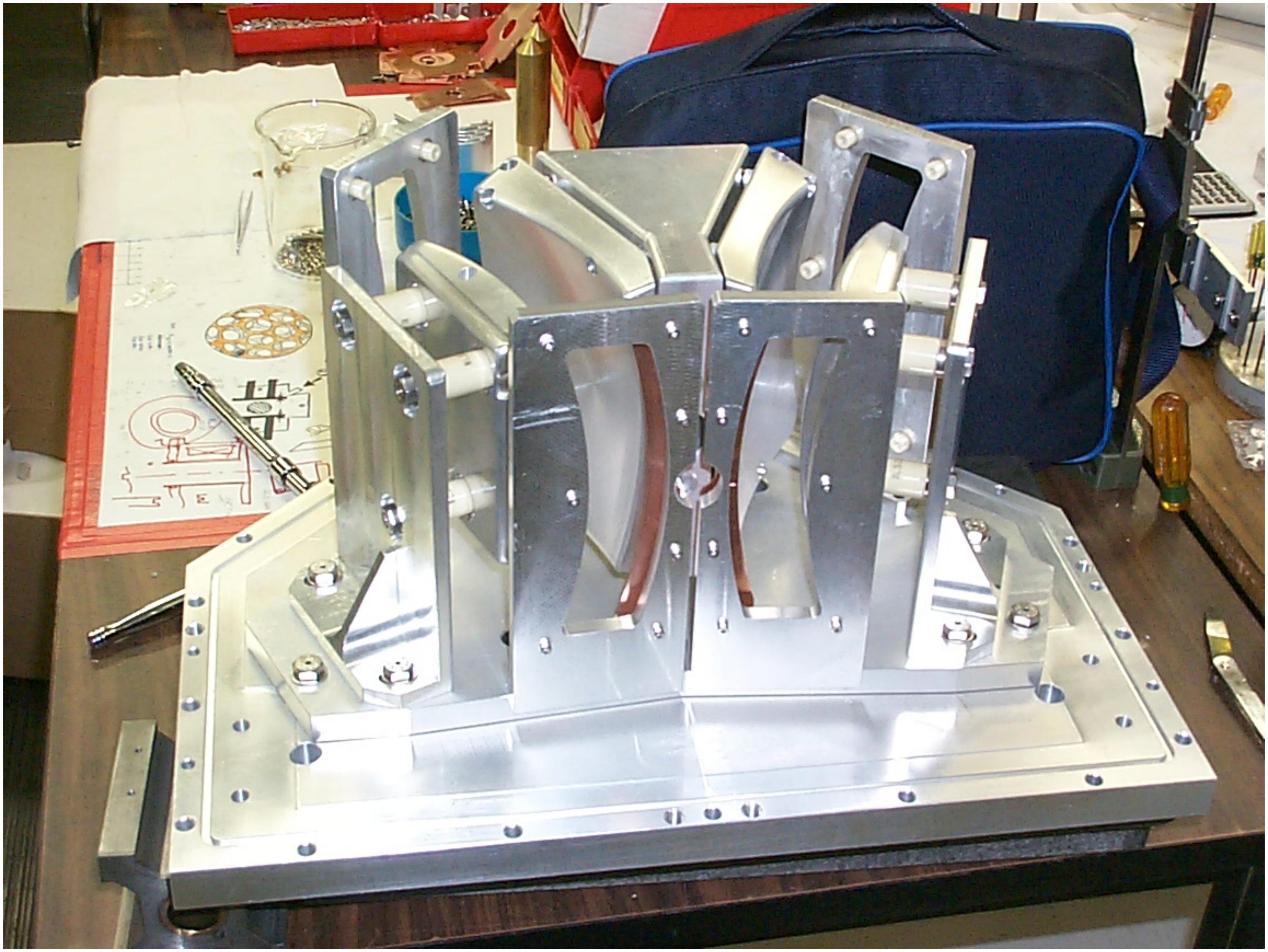
Bending: For radius ρ, require field E_r:

$$qE_r = \frac{mv^2}{\rho}$$

$$E_r = \frac{2V_E}{\rho}$$

So for again a 30keV beam, choose a 10 inch radius, electric field is only 6 kV/in or 0.24 MV/m.

The electric field “follows” the beam direction, and so electrodes must be curved: cylindrical if we allow off-plane particles to not be focused. However, just as with magnetic bends, the bending field focuses strongly in the plane. If we want to focus in both planes equally together, then the electrode shape is spherical.

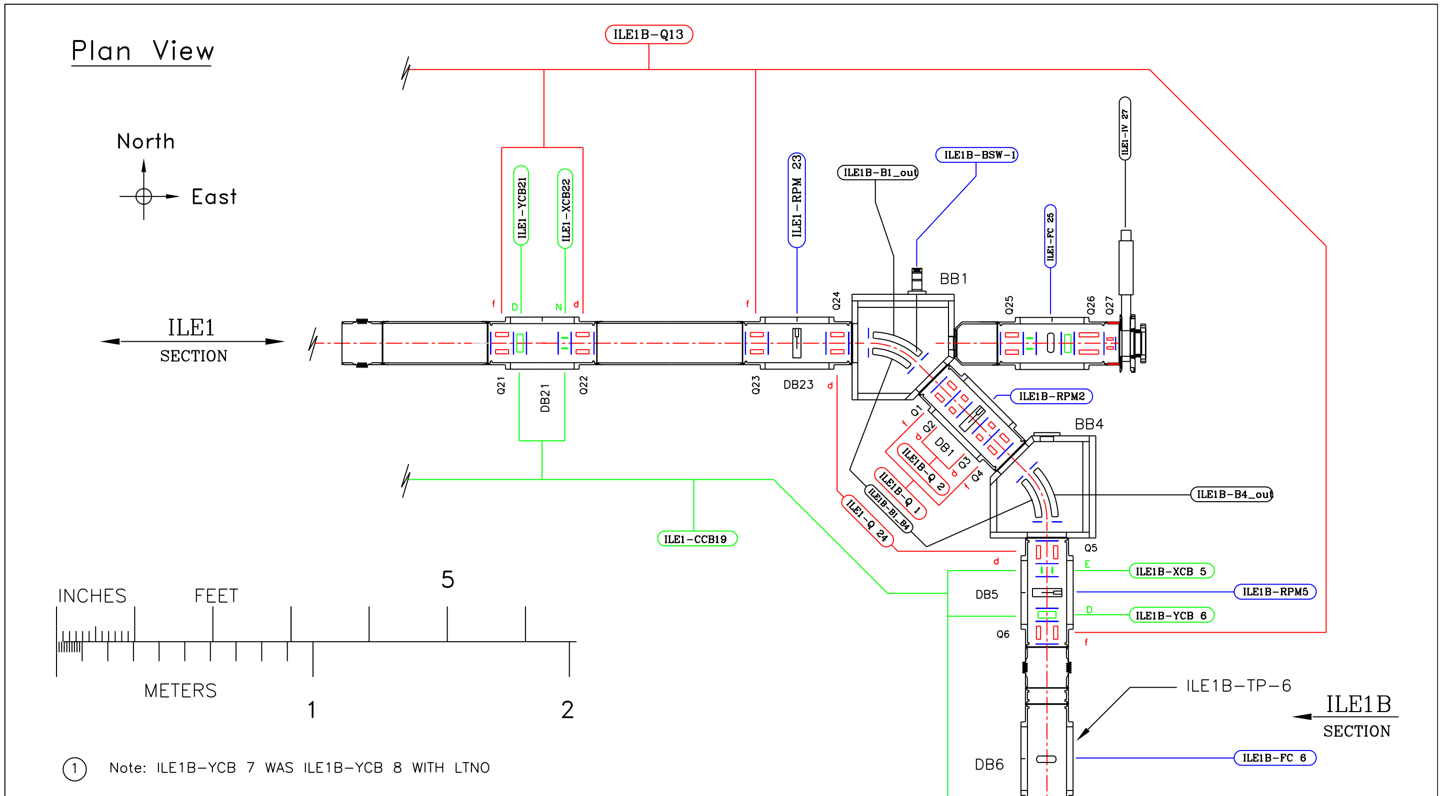


Quadrupoles are intrinsically linear, but bends are not: spherical $\Rightarrow E_r \propto 1/r$. This constrains beam size $\ll \rho$. Thus, quad apertures can be as large as we like, but bend apertures (for given ρ) cannot.

Scaling

Notice that in both cases, potentials depend only upon energy (actually on the potential difference between source and beamline), not at all upon mass. This means that any co-moving particles same energy per charge but of whatever mass will act exactly as the desired particles.

To eliminate other species, need magnetic fields.



(1) Note: ILE1B–YCB 7 WAS ILE1B–YCB 8 WITH LTNO

REV	DATE	REVISION DESCRIPTION	BY	APP'D
2	28/05/08	Added 4in Nipple to DB7	RD	
1	26/05/2008	Removed ILE1B–FC–5, added FC–6 & TP–6	RD	

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ILE1B Section – Optics

GPS–2 RELOCATION

DRAWN	APPROVED	DWG NO.	REV
Ray Dube		ISK4461D	2
DATE 6 May 2008			

Thermal μ Beams

What is thermal angle $x' = \frac{P_x}{P_0}$?

$\frac{P_x^2}{2m} \approx 0.025$ eV, $\frac{P_0^2}{2m} \approx 30$ keV,

$$x' \approx \sqrt{\frac{0.025}{30000}} \sim 10^{-3},$$

or, 1 mradian. Say the "source" is 200 mm in diameter (huge), then emittance is 1mrad $\times$ 100mm; $\epsilon \sim 100\,\mu$m (usually referred to as "100πmm-mrad"). Optically, Twiss β-function $\sim$ 1metre, so beam size is

$$2x_{\text{rms}} = 10\,\text{mm}$$

Quite reasonable even for this "huge" emitter 200mm in diameter.

Solenoidal fields

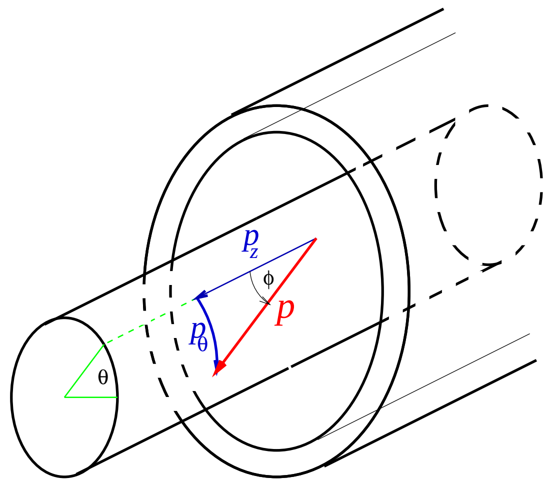
But whenever a beam of charged particles is created inside a solenoidal field and subsequently extracted, there is a loss in beam quality.

The canonical momentum of a particle of charge q is

$$\vec{P} = \vec{p} + q\vec{A}, \tag{1}$$

where $\vec{p}$ is the usual momentum, and $\vec{A}$ is the magnetic field's vector potential. In a region with no electric field the Hamiltonian is independent of position and so the canonical momentum is conserved. If a particle originates inside a magnetic field and travels to a point well outside of it, it receives a change of (ordinary) momentum

$$\Delta\vec{p} = q\vec{A}. \tag{2}$$



For a solenoid with magnetic field B_0, $\vec{A}$ has only a component in the azimuthal (θ) direction

$$A_\theta = \frac{r}{2} B_0. \tag{3}$$

So on exiting the solenoid at a radius r, the particle gets an azimuthal kick:

$$\phi = \frac{\Delta p_\theta}{p_z} = \frac{qr}{2p_z} B_0 \tag{4}$$

It is often convenient to express the momentum as a 'rigidity' $B\rho = p/q$. Then the angular kick is expressed very simply as

$$\phi = \frac{r}{2\rho}, \tag{5}$$

where ρ is the radius of curvature that the particle would have if it were travelling with momentum p_z orthogonal to the field B_0.

Example: A muon at 30keV has $B\rho = 0.0084$ T-m. In a field of 0.3 Tesla, $\rho = 28$ mm. So a beam that has radius 10 cm, is completely screwed; angles are 1800 mradian, and emittance becomes $18000\,\pi$mm-mrad ($18000\,\mu$m). Even for a 1 cm radius beam, $\epsilon = 1800\,\mu$m.

This is far larger than the thermal emittance and would make such a low energy beam impossible to transport.

On a more fundamental level, think of the Hamiltonian $H(r, \theta, z; t) = E$:

$$(E - q\Phi)^2 = (m_0 c^2)^2 + |\vec{P} - q\vec{A}|^2 c^2 \tag{6}$$

To lowest order,

$$A_\theta = \frac{r}{2} B(z) \tag{7}$$

has no θ-dependence. Therefore H has no θ-dependence. Therefore angular momentum is conserved.