Use of the Strong Collision Model to Calculate Spin Relaxation

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Abstract

In the classic μSR paper [see Hayono et al., Phys. Rev. B **20**, 850 1979)] on zero field μSR the strong collision model was used to determine the time evolution of the muon spin polarization in situations where there are random fluctuations in the muon's local environment. Its main advantage is the simplicity and intuitive interpretation. It has been used to model the spin dynamics of the muon or muonium under a variety of circumstances. In this paper we examine the model in a long time limit, where t is much greater than the correlation time τ_c for fluctuations in the local environment. This situation is particularly relevant to β-NMR but may also be useful in μSR. We show the longitudinal polarization decays at a rate which has a simple analytic expression involving a sum of Lorentzians.

Application of Strong Collision model

Prior to any collision we assume the expectation of I_z of the probe (muon or nucleus) evolves according to a static spin Hamiltonian and an initial density matrix $\rho(0)$

$$p_s(t) = \langle I_z \rangle = \sum_{jk} \langle e_j | \rho(0) \rangle \langle \rho(0) | e_k \rangle \langle e_j | I_z | e_k \rangle e^{-i\omega_{jk} t} \quad \text{where} \quad H|e_j\rangle = e_j|e_j\rangle \qquad \text{Eq.1}$$

We assume that after a "collision" all information about the environment is suddenly lost and that part of the density matrix associated with the environment is restored to its initial unpolarized state. Basically this means that after a collision the polarization of the probe in a longitudinal field continues to evolve as at t=0, but with a reduced amplitude. Thus after each collision a small amount of polarization is lost. This is illustrated in Fig. 1 which shows the time evolution of the polarization after a few collisions in the simplest case where the static polarization function in a longitudinal field is .

$$p_s(t) = 1 - a + a\cos\omega t.$$

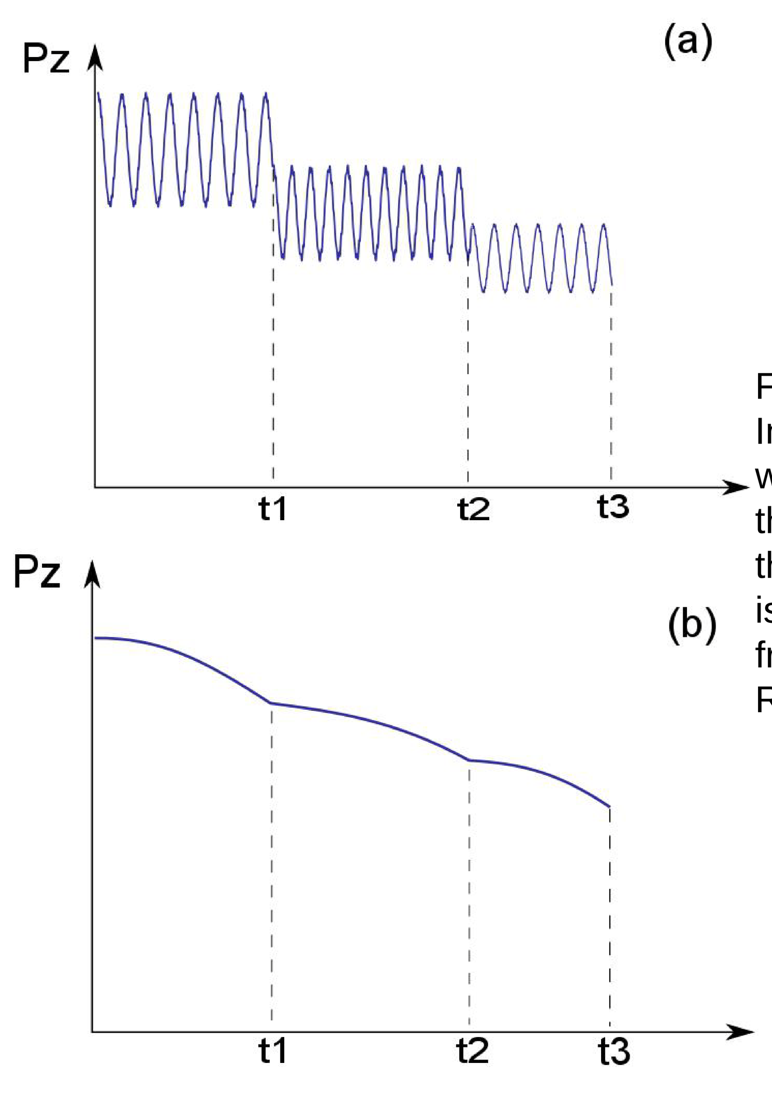


Fig. 1 (a) Schematic of the polarization In the strong collision model in the limit where the collision rate is much less than the oscillation frequency $\nu << \omega$ (b) in the opposite limit where the collision rate is much greater than oscillation frequency. [see K.H. Chow, B. Hitti and R.F. Kiefl **51**, 137 (1998).]

Spin relaxation in the long time limit

Typically in μSR the strong collision model is used to "dynamicize" a static polarization function. The classic example is to simulate the time evolution of spin polarization function for a muon in a crystal where the magnetic dipolar fields from the surrounding nuclear spins change suddenly as the muon hops. In this case the static polarization is given by the familiar function derived and made famous by Kubo and Toyabe. The problem is tractable but in general requires a complicated Laplace transform that must be done numerically. In this paper we point out that in the long time limit after many collisions the dynamic polarization evolves according to a simple exponential with a relaxation rate which consists of a simple sum of Lorentzians -- one for each frequency in the expression for $p_s(t)$. This situation is not uncommon in μSR and almost always the case in β-NMR where the radioactive lifetime is much longer.

Our central result comes from the assertion that after many collisions the ensemble averaged dynamic polarization at time t, P(t), decreases at a rate given by the product of P(t), the collision rate ν and the average polarization loss from the first collision.

$$\frac{dP(t)}{dt} = -P(t)\nu\langle 1 - p_s \rangle$$

where the average loss in polarization from the first collision is given by:

$$\langle 1 - p_s \rangle = 1 - \nu \int_0^\infty \exp[-\nu t'] p_s(t') dt'$$

In general the static polarization function can be written as a sum of cosines

$$p_s(t) = \sum_i a_i \cos\omega_i t$$

so that the mean polarization loss from the first collision:

$$\langle 1 - p_s \rangle = 1 - \sum_i a_i \frac{\nu^2}{\nu^2 + \omega_i^2} = \sum_i a_i \frac{\omega_i^2}{\nu^2 + \omega_i^2}$$

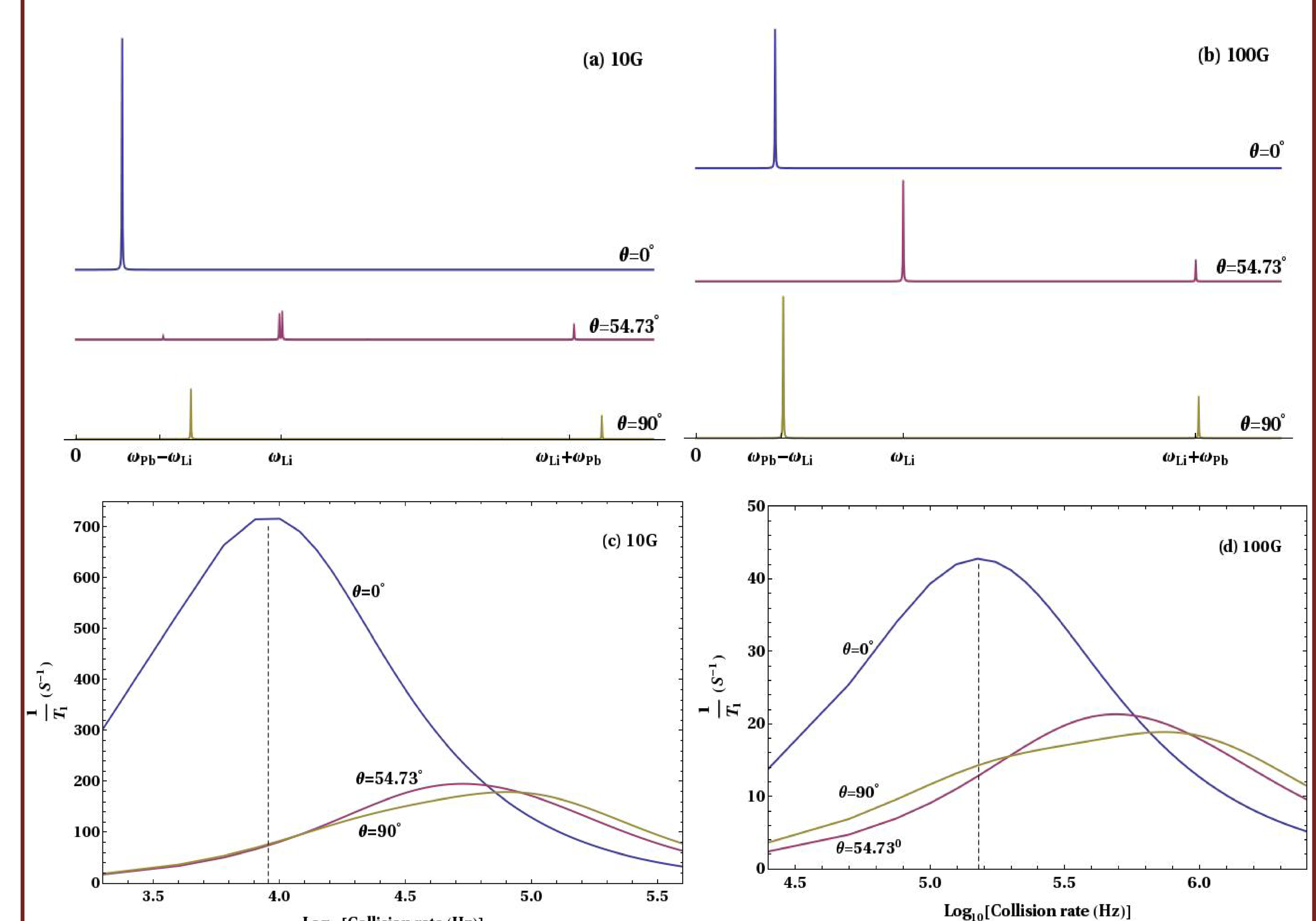
This leads to a particularly simple result for the relaxation rate

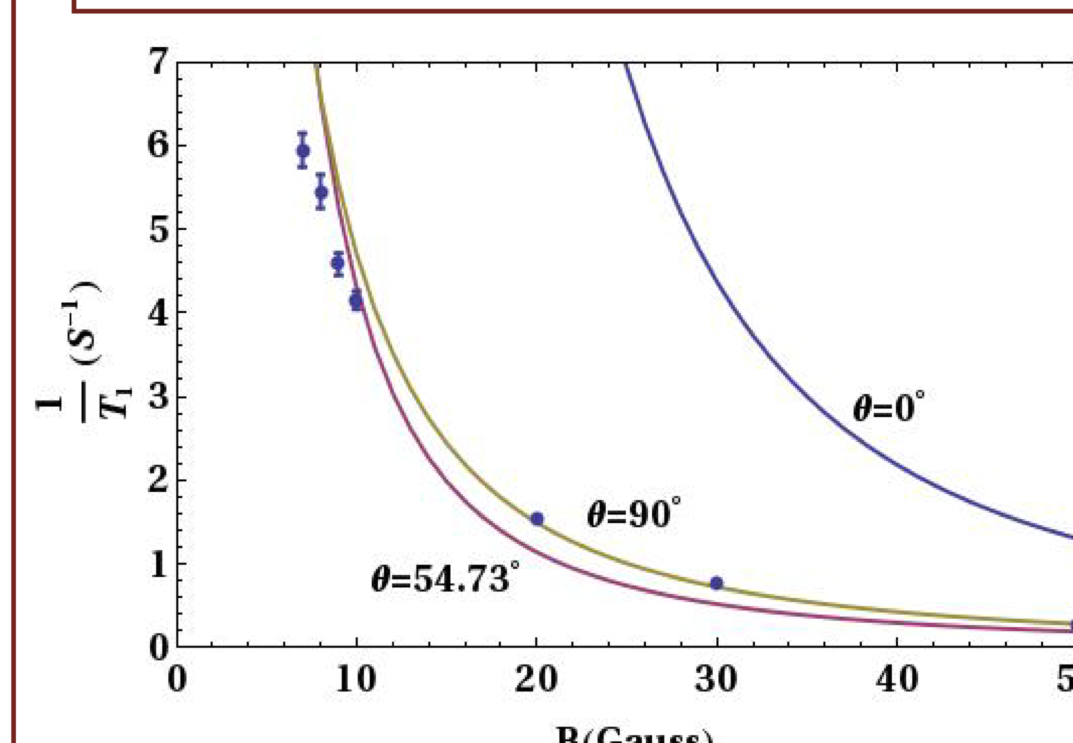
$$\frac{1}{T_1} \equiv -\frac{1}{P(t)}\frac{dP(t)}{dt} = \nu\langle 1 - p_s \rangle = \nu \sum_i a_i \frac{\omega_i^2}{\nu^2 + \omega_i^2} \qquad \text{Eq. 2}$$

The interpretation is remarkably simple. Each frequency in the static polarization function contributes a single Lorentzian to the expression for $1/T_1$. For a fixed magnetic field the contribution from any given frequency peaks as a function of ν when $\nu = \omega_i$ whereas the magnitude of each peak equals $a_i\omega_i/2$. This is the familiar T_1 minimum affect from NMR theory. The peaks are generally broad and typically unresolved giving rise to a single T_1 minimum at some intermediate value of ν (see figure on next panel). Since the field is being applied along the initial polarization direction the oscillation amplitudes, a_i, decrease as $1/B^2$ in high field so the resulting peaks in $1/T_1$ peak decrease roughly as 1/B. The exception for this is near level crossings. Note also the field dependence of $1/T_1$ for a fixed value of ν also depends on the static frequency spectrum in $p_s(t)$ and will fall as $1/B^2$ in high magnetic fields, except near level crossings.

Spin 2 ^{8}Li coupled to a single spin ½ ^{207}Pb

Consider a spin 2 ^{8}Li nucleus coupled to a single spin ½ ^{207}Pb nucleus with magnetic dipole interaction strength of 3600 s^{-1} corresponding to an effective dipolar field of about 1G acting on the ^{8}Li. Top panels in the figure below show the static frequency spectrum in a longitudinal fields of 10G and 100G for three orientations of field with respect to the internuclear direction. As expected the spectrum of frequencies is more complicated in low field. However in a high field where the dipolar interaction is a small perturbation the frequencies are close to $\omega_{Pb} - \omega_{Li}$, ω_{Li} and $\omega_{Pb}+\omega_{Li}$ with angular dependent magnetic amplitudes. The bottom two panels show the spin relaxation rate versus collision rate for 10G and 100G respectively. The peak in each case are due to a multiple T_1 minimum effect which occur when $\nu=\omega_i$ with amplitude $a_i\omega_i/2$ as predicted from Eq. 2.





Magnetic Field Dependence of $1/T_1$. Data points are the spin relaxation rate of ^{8}Li measured in Pb as function of magnetic field. The spin relaxation is attributed to fluctuations in the nearest neighbor ^{207}Pb spins from host T_2 and T_1 processes. The curves are simulations using the strong collision model for different orientations of the applied field with respect to the internuclear direction using Eq. 2, with ν =500Hz.

Conclusion:

We have shown that the strong collision model can be used to calculate spin relaxation rates in the long time limit i.e. after many collisions. The form of the relaxation rate is particularly simple consisting of a sum of Lorentzians -- one for each frequency in the static polarization function in a longitudinal field. There are many applications in β-NMR and μSR .